

Problems in Randomization of Online Question Banks (Case Studies)

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Abstract

At the University of Pécs (Hungary) mathematics courses are the basis for programs such as engineering and computer science. An online question bank was developed for university students to practice mathematical problem-solving and to use for exam assignments.

Computer-aided test and assessment is widely used to support the teaching and learning of mathematics.

In the research some aspects of question design were asked about the new possibilities of randomizing the question::

- 1. What type of randomization is useful to introduce, and what are the limits of randomization?*
- 2. Why is it sometimes so dangerous, what are the benefits, and what are the limits of it?*
- 3. What type of math problems can be solved using random variables?*
- 4. How can we ensure equal difficulty for the variation of questions?*

In our paper, some case studies of the task design are presented. The examples are from courses in mathematics at BSc and MSc levels, as well.

1. Introduction

During the pandemic lockdown, the teachers in schools and lecturers at the universities turned to formative and summative assessments [1], [2], [3]. Nowadays assessments are the focus of didactical research [4], [5]. The well-developed assessments (assessment with digital technology or assessment through digital technology) [6] are well-usable tools to get more accurate feedback about the student's mathematical knowledge, helping lecturers guide and monitor the students' progress [7], [8], [9]. At the Department of Mathematics, the University of Pécs (UP), Faculty of Engineering and Information Technology during the pandemic different types of questionnaires were introduced (first of all to avoid cheating), The MOODLE LMS can handle most digital technology-based assessment systems, but there are only few which manages the mathematical equivalences (STACK based on Maxima [10], MATHLAB Grader based on MATHLAB [11], GeoGebra assessment [12], MÖBIUS earlier MAPLE T.A. based on MAPLE, etc). Because we have had experience using Maple computer algebra system (CAS) during math courses for a long time [13], [14] we have chosen the Maple-based Möbius test and assessment system [15], [16]. Assessments with digital technology are used not only for online courses but conventional (face-to-face) courses as well.

The Möbius TA was introduced to practice mathematical problem solving (formative assessment) and to use for exam situations (summative assessment) as well.

In this paper, there is a discussion about the emerging problem during the development of question banks in online test and assessment system. Our didactical questions are about the ideas of randomization and evaluation.

There are so many ways to randomize in a test. In a conventional test system, there are

- randomly generated quizzes from pools of questions
- random orders in
 - o blocks
 - o questions
 - o possible answers in multiple choice questions.

In a CAS-based system, the questions could be generated with random variables, and the students see different variants of a question.

In our research we are looking for answers to the following questions:

- What type of randomization is useful to introduce, and what are the limits of randomization?
- Why is it sometimes so dangerous, what are the benefits, and what are the limits of it?
- What type of math problems can be solved using random variables?
- How can we ensure equal difficulty for the variation of questions?

In the next chapters, some case studies of the task design and the evaluation changes are shown. The examples are from courses in mathematics for STEM courses, at BSc and MSc levels, as well.

2. Simple randomization

The first example is for freshmen students. It is a simple problem, with six random values which are simple integers; (easy calculation is necessary and because from a didactical point of view, no calculator or computer algebra is allowed, the assessment system was used in proctored mode). Two cases (*maximum*, *minimum*) increase the variability of the two unknowns (Figure 1, Figure 2, Figure 3). There are two conditions, a and b are not equal to 0. a is not zero to have an absolute value function, and b is not zero for the questions to be of similar difficulty.

Find the values of the missing parameters if the form of the function is

$$f(x) = a|x| + b,$$

the 'minimum value' of the function is equal to -4, and

$$f(-5) = 21$$

The domain of the function is the real number set.

$a =$ $\quad b =$

Figure 1: Example 1. Simple random question, in the question's preview

Find the values of the missing parameters if the form of the function is

$$f(x) = a|x| + b,$$

the \$m of the function is equal to \$b, and

$$f($x0) = $c$$

The domain of the function is the real number set.

$a =$ $\quad b =$

Figure 2: Example 1. Simple random question, question's text

```

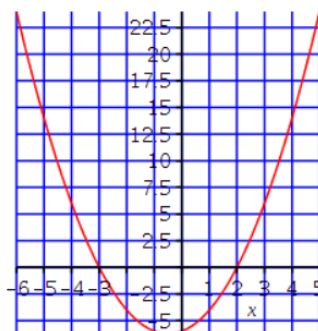
$a=range(-5,5);
condition: ne($a,0);
$b=range(-5,5);
condition: ne($b,0);
$x0=range(-5,-1);
condition: ne($x0,0);
$c=maple("$a*abs($x0)+($b)");
$m=maple("if $a>0 then `minimum value` else `maximum value` end if");
$m_szep=maple("MathML[ExportPresentation]($m)");

```

Figure 3: Example 1. Simple random question, question algorithm

3. Randomization of figure

Example 2. is about the random figure in a question. This question is for freshmen students, as well, they had to read the parameters from the figure. Conditions were: the whole square was to be avoided, the figure axes must be dynamic ones, and the intervals on the axes depend on the function's definition. Three answers are needed, so the points of the question could be partial (Figure 4). The commutativity is handled in the grading part. Remark: the disadvantage of the display is, if the answer is negative, there are no parentheses for it. Maybe it's not confusing because there's a relatively large space separating the answer from the text of the question. From a didactical point of you, the quality of the figure is very important, students have to understand the connection between the meaning of zero points and the existence of minimum or maximum and the asked values. Because of the dynamical visualization, the condition in the question is that $b < c$.



Find the formula for the above-plotted parabola in factorized form

y= · (x+) · (x+)

```

$m=range(-1,1);
$b=range(-3,-1);
$c=range(1,3);
condition: ne($m,0);
condition: ne($b,0);
condition: ne($c,0);
condition: ne($c,$b);
$f=maple("$m*(x-($b))*(x-($c))");
$f_szep=maple("MathML[ExportPresentation]($f)");
$hv1=maple("-( $b )");
$hv2=maple("-( $c )");
$x1=maple("( $b )-3");
$x2=maple("( $b )+3");
$y1=maple("if $m=1 then $c-4 else $c-($c)/2 end if");
$y2=maple("if $m=1 then ($c)+4 else 1 end if");
$wave1=plotmaple("plot($f,x=($b)-3..($c)+3,'color'=red,
axis = [thickness=1,gridlines = [10,colour = blue]]),
plotdevice='jpeg', plotoptions='width=250,height=250'");

```

Figure 4: Example 2. Random figure

4. Randomization of wrong answers in multiple choice and theoretical question

Example 3. shows, how the random variables are connected, and how wrong answers depend on the correct ones. The wrong answers are calculated from the correct ones, using the *ListTools* Maple package. The question is not about the solutions of the equation, but only the number of solutions on a closed or open or half-closed, half-open interval. From didactical points it is a complex question, it connects different knowledge backgrounds, the meaning of interval, and the number of the solution of the trigonometrical equation. The variables are reduced depending on whether k is odd or even, and the equations are $\sin(kx) = 0$ or $\cos(kx) = 0$ (Figure 5). Planning is not so easy: the designer must be an expert on math didactic, a perfect Maple user, and a Möbius script writer. That's why most of the teachers are using only easily programable questions or well-prepared questions from other question banks.

```

How many roots does the equation $tridgeq have on the interval $int ?

 $correct
 $wrong1
 $wrong2
 $wrong3
 $wrong4

How many roots does the equation  $\cos(3x) = 0$  have on the interval  $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$  ?
O5 O4 O2 O6 O3

$K = range(2,3); # sin(k*x), oos(k*x)
$var1 = rint(2); # left side is open or closed
$var2 = rint(2); # right side is open or closed
$bo = switch($var1, "\biggl[", "\biggl[");
$jo = switch($var2, "\biggr]", "\biggr");
$ver = rint(2);
#interval J=[Pi/2;3Pi/2[, if k is odd, if k is even and k/2 is odd,
#then [Pi/4;5Pi/4[, if k/2 is even [Pi/(2k);Pi/(2k)+Pi[
$int = switch($ver, "0;\pi", if($k-2, "\frac{\pi}{2};\frac{3\pi}{2}",
"\frac{\pi}{4};\frac{5\pi}{4}");
$int = "\(" $bo $int $jo \)";
$tridgeq = switch($ver, "\(\sin($k)x=0 \)", "\(\cos($k)x=0 \)");
$h = -1+$k+$var1+$var2; # correct answer
# 4 wrong answers
$T = min($h,rint(5)); # using Maple built in package
$s = maple("with(ListTools):
L:= [seq($h-$T+i,i=0..4)];
L:=Rotate(L,$T);
L[ ]");
$correct = switch(0,$s);
$wrong1 = switch(1,$s);
$wrong2 = switch(2,$s);
$wrong3 = switch(3,$s);
$wrong4 = switch(4,$s);
    
```

Figure 5: Example 3. Multiple choice question

If the question is a theoretical one from the differential equation (Example 4.), only the coefficients and exponent values manage the correct answer. The possible answers are on the lists, but they are not static, the random coefficient, exponent, and the function on the right-hand side manage them. The simple lists manage the answer, but the correct answer depends on the equation (Figure 6, Figure 7).

Let $2 \left(\frac{d^2}{dx^2} y(x) \right) + 3 \left(\frac{d}{dx} y(x) \right) + y(x)^2 = \cos(x)$

This ODE is

(Click for List) ▾	(Click for List) ▾	(Click for List) ▾
(Click for List) "second order" "first order"	(Click for List) ▾ (Click for List) "non-linear" linear	(Click for List) ▾ (Click for List) homogenous inhomogenous

Figure 6: Example 4. Theoretical question

```

$a=rint(3);
$b=rint(4);
condition:=ne($a,$b);
$c=range(1,2);
$f=switch(rint(3),y*cos(x),cos(x),0);
$de:=maple("($a)*diff(y(x),x$2)+($b)*diff(y(x),x)+y(x)^( $c)=$f");
$de_szep:=maple("MathML[ExportPresentation]($de)");
$hv1:=maple("if $a=0 then `first order` else `second order` end if");
$rv1:=maple("if $a=0 then `second order` else `first order` end if");
$hv2:=maple("if $c=1 then `linear` else `non-linear` end if");
$rv2:=maple("if $c=1 then `non-linear` else `linear` end if");
$hv3:=maple("if $f=cos(x) then `inhomogenous` else `homogenous` end if");
$rv3:=maple("if $c=cos(x) then `homogenous` else `inhomogenous` end if");
    
```

Figure 7: Example 4. Conditions in theoretical question

5. Randomization using a self-made Maple package

Sometimes developers have to create their own Maple package to design a question of equal difficulty for everyone. Example 5. is a linear algebra question, asking about the reduced row echelon form from a linear system and the number of solutions. In this situation, the correct answer to the multiple-choice question depends on the rank of the system. The developed *LinAlgTools* package [15] solve an everyday problem for teaching linear equation system. If there are arbitrary coefficients in the system, the result of the row operation process sometimes causes miscalculations because of the fractions, and the students can not focus on the meaning of the results (mainly the number of solutions and the explanation). Using this new Maple package the coefficients are integers, but the solution types could be different. To avoid mistyping there is some instruction for input written in red (Figure 8), this is the way how the users are helped to have the input correctly.

Find the solution using Gaussian elimination

$$\begin{aligned}
 2x - 10y - 14z &= 28 \\
 5x - 23y - 33z &= 70 \\
 -4x + 23y + 29z &= -48
 \end{aligned}$$

A) The augmented matrix of the LES is

The form of the matrix is $\text{Matrix}([[a_{11}, a_{12}, a_{13}, a_{14}], [a_{21}, a_{22}, a_{23}, a_{24}], [a_{31}, a_{32}, a_{33}, a_{34}]])$

B) The reduced row echelon form is

C) The LES has

'no solution'

'unique solution'

'infinite number of solutions'

Correct

Your Answer: $\text{Matrix}([[1,0,0,6],[0,1,0,4],[0,0,1,-4]])$

Correct Answer: $\begin{pmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -4 \end{pmatrix}$

```

$x=range(-5,5);
$y=range(-5,5);
$z=range(-5,5);
$r=switch(rint(2),2,3);
$a=range(1,2);
$Q:=maple(" M:=if $a=1 and $r=2 then LinAlgTools:-SpecialMatrix([3,3,$r],aug,nosol)
else LinAlgTools:-SpecialMatrix([3,3,$r],aug) end if:
S:=LinearAlgebra:-GenerateEquations(M,[x,y,z]):

R:=LinearAlgebra:-ReducedRowEchelonForm(M);
convert~([M,S[1],S[2],S[3],R],string),
libname=/web/Ptemik001/Public_Html/Packages/LinAlgTools.mla");
$M:=switch(0,$Q);
$M_szep:=maple("MathML:-ExportPresentation($M)");
$S1=switch(1,$Q);
$S2=switch(2,$Q);
$S3=switch(3,$Q);
$R=switch(4,$Q);
$R_szep:=maple("MathML:-ExportPresentation($R)");
$L:=maple("$R[3]");
$L_szep:=maple("MathML:-ExportPresentation($L)");
$R34:=maple("$R[3,4]");
$S1d:=maple("MathML:-ExportPresentation($S1)");
$S2d:=maple("MathML:-ExportPresentation($S2)");
$S3d:=maple("MathML:-ExportPresentation($S3)");
$hv:=maple("if $R[3,3]<>0 then `unique solution` elif $R[3,4]=0 then `infinite number of solutions`
else `no solution` end if");
$rv1:=maple("if $R[3,3]<>0 then `no solution` elif $R[3,4]=0 then `unique solution`
else `infinite number of solutions` end if");
$rv2:=maple("if $R[3,3]<>0 then `infinite number of solutions` elif $R[3,4]=0 then `no solution`
else `unique solution` end if");
    
```

- (1) \$hv fixed
- (2) \$rv1 fixed
- (3) \$rv2 fixed

Figure 8: Example 5. Using self-made package

6. Backward thinking – Maple and precise mathematics

As tutors, we know from our earlier practice developing a good question, sometimes is useful to think backward. In Example 6. the question is about the indefinite integral of rational functions, using partial fractions. Inside the script, the improper fraction is given as a sum of a polynomial and a proper fraction but in factorized form. But in the question text, the function appears in expanded form. It is necessary to have the same number of factors, the same number of polynomial terms, and proper terms (Figure 9) to have equivalent difficulty in the answer.

Divide the numerator into the denominator to get a polynomial plus a proper fraction, and determine the antiderivative

$$f(x) = \frac{4x^3 + 15x^2 + 7x - 10}{(x+1)(x+3)}$$

Polynom part $\frac{4x^3 + 15x^2 + 7x - 10}{(x+1)(x+3)} =$

Proper fraction $\frac{4x^3 + 15x^2 + 7x - 10}{(x+1)(x+3)} =$

The indefinite integral $\int \frac{4x^3 + 15x^2 + 7x - 10}{(x+1)(x+3)} dx =$ `|x| => abs(x)`

```

condition: ne(%c,0);
condition: ne(%a,0);
condition: ne(%d,0);
condition: ne(%e,0);
condition: ne(%b,1);#because of ln(abs(x))
condition: ne(%b,0);
condition: ne(%e,($g));#two different quotients
$f:=maple("factor(%a*x+(%b)+(%c)/(x+(%e))+(%f)/(x+(%g)))"); #reverse thinking
$sol:=maple("convert($f,parfrac");#reverse thinking

$poli:=maple("(op(1,$sol))+op(2,$sol)");#the polynomial has 2 terms
$proper:=maple("(op(3,$sol))+op(4,$sol)");
    
```

Figure 9: Example 6. Backward thinking

Another possibility to define rational function is to use Maple *randpoly* command, but the result could be varying difficulties and forms (Figure 10), so it was neglected in the question.

<pre> Random polynomial -> > num := randpoly(x, coeffs = rand(-2..2), degree = 4) num := x^3 - 2x^2 - 2x - 2 > denom := randpoly(x, dense, coeffs = rand(-2..2), degree = 2) denom := -x^2 - x > g := num/denom g := (x^3 - 2x^2 - 2x - 2)/(-x^2 - x) > convert(g, parfrac) -x + 3 - 3/(x+1) + 2/x </pre>	<pre> Random polynomial -> > num := randpoly(x, coeffs = rand(-2..2), degree = 4) num := x^4 + 2x^3 - 2x^2 + x - 2 > denom := randpoly(x, dense, coeffs = rand(-2..2), degree = 2) denom := -x^2 - 2x + 2 > g := num/denom g := (x^4 + 2x^3 - 2x^2 + x - 2)/(-x^2 - 2x + 2) > convert(g, parfrac) -x^2 + (-x + 2)/(x^2 + 2x - 2) </pre>
--	--

Figure 10: Example 6. Different results using Maple *randpoly* command

In this situation, as strict teachers, we are not satisfied with the indefinite integral for $\frac{1}{x}$ as $\ln(x)$, which is given by Maple but we are waiting for $\ln|x| + C$. So it has to be realized which part of the integral has *ln* form to transform it into the correct answer form. (Figure 11.)

```

$intsol:=maple("int(($f),x)");#integral in Maple form not ln(abs) no C
$op21:=maple("op(2,op(1, $intsol))");
$op22:=maple("op(2,op(2, $intsol))");
$op23:=maple("op(2,op(3, $intsol))");
$op24:=maple("op(2,op(4, $intsol))");
$i1sol:=maple("if $op21 = ln(x+($e)) then ln(abs(x+($e)))
elif $op21 = ln(x+($g)) then ln(abs(x+($g)))
else $op21 end if"; # int(1/x,x)=ln(x) and not ln(abs(x))+C
$i2sol:=maple("if $op22 = ln(x+($e)) then ln(abs(x+($e)))
elif $op22 = ln(x+($g)) then ln(abs(x+($g)))
else $op22 end if";
$i3sol:=maple("if $op23 = ln(x+($e)) then ln(abs(x+($e)))
elif $op23 = ln(x+($g)) then ln(abs(x+($g)))
else $op23 end if";
$i4sol:=maple("if $op24 = ln(x+($e)) then ln(abs(x+($e)))
elif $op24 = ln(x+($g)) then ln(abs(x+($g)))
else $op24 end if"; #the order is not always appropriate
$isol:=maple("(op(1,op(1, $intsol)))*($i1sol)+(op(1,op(2, $intsol)))*($i2sol)+
(op(1,op(3, $intsol)))*($i3sol)+(op(1,op(4, $intsol)))*($i4sol)+C");

```

Incorrect	
Your	No answer
Answer:	
Correct	$2x^2 - x - 3 \ln(x + 1) + 2 \ln(x + 3) + C$
Answer:	

Figure 11: Example 6. Handling integral $1/x$

7. Using CAS to have the right answer

It is a question to MSc students, they are allowed to use CAS during the exam. The students are familiar with Maple commands, the well-prepared worksheets were introduced in the face-to-face classes, which were held in a computer lab. In the exam problem, there were randomized data, but they are defined in connection to each other (Figure 12). On the preview page, the correct answer is seen, but only on a figure, which helps to have the correct answer (Figure 13). Randomized data, randomized figures, and intervals were used.

The table below shows the relationship between the average speed of a car (km / h) and the stopping distance (m) for that speed

X=\$X_szep
Y=\$Y_szep

Determine - using the least-squares method - whether a linear, quadratic or quadratic function relationship between the two quantities approximates the relationship between the data with the least error.

Use Maple or Excel for your calculation

```

$X:=maple("[5.4, 9.6, 14.9, 19.8, 25.6, 29.4, 35.3, 39.7]");
$Z:=maple("[5.6, 11.8, 22.4, 39.9, 53.9, 71.9, 90.3, 122.3]");
$Y:=maple("for 1 to 8 do
    R[1] := rand(-8 .. 8);
    YY[1] := $Z[1] + evalf(R[1]()/10, 3);
end do:
[seq(YY[j], j = 1 .. 8)]");

```

Figure 12: Example 7. Randomized data with condition

8. Evaluation

It is an interesting and important issue to investigate the result of exams with and without an online test system. We had an experiment during the Covid 19 when our students had remote exams. The course was Engineering Mathematics 2 (BSc, Semester 2). There were two groups, in the first group the students were from the Electrical and Civil Engineering (EE and CE) program, and in the second one the students were from Computer Science Engineering (CSE) program. The same classes were held for both groups, they had two parts, one part was the lecture, and the other is the practice. During the practice part, the students got an assignment with non-randomized questions. They had 45 minutes to solve the problems, it was allowed to collaborate. Every week they had an assignment with randomized questions to practice the discussed topics alone. During the semester there was two homework to be submitted on Möbius, with randomized questions. The difference was in the exam situation. The group CSE had the homework and exam on Möbius with

randomized questions, and the control group's (CE and EE) homework was on Möbius, but during the exam, the 'solve-it on paper-then-upload-it ' method was used.

Linear approximation: $y =$ $\cdot x +$

The sum of the squares of the differences:

Quadratic approximation: $y =$ $\cdot x^2 +$ $\cdot x +$

The sum of the squares of the differences:

Third ordered approximation: $y =$ $\cdot x^3 +$ $\cdot x^2 +$ $\cdot x +$

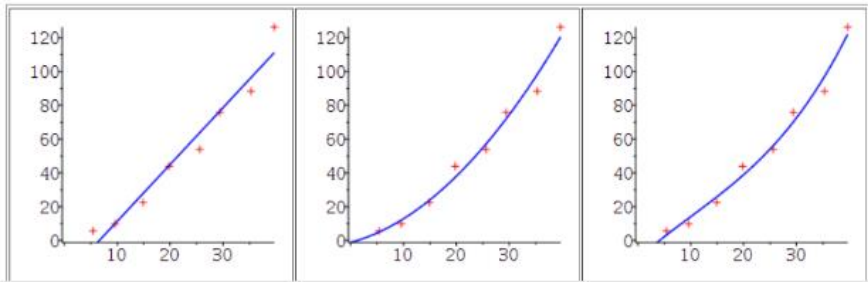
The sum of the squares of the differences:

The best approximation

second ordered

first ordered

third ordered



```
$hv=maple("if ($diff_lin<$diff_square) and ($diff_lin<$diff_cube) then 1 elif $diff_cube<$diff_square then 3 else 2 fi");
```

Figure 13: Example 7. Calculation in Möbius with help of CAS

The next example from the test highlights the difference between the two forms of the exam. For the paperwork, one question was:

Find the regions of integration and evaluate the double integral over the region bounded by $y = 4x, y = x, x = 2, x = 4$ lines, if $f(x, y) = x - 4y$.

A similar question on Möbius is seen in Figure 14.

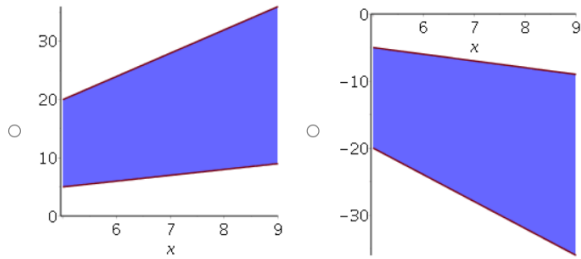
We expected that Möbius question helps the students to go through to the solution, but unfortunately, it was not true. Comparing the two groups we found that there is a much higher degree of correlation between the homework average and the exam average for CE and EE group than for CSE. Unsurprisingly, identical (identically flawed) solutions were prevalent among CE and EE group. One explanation of this phenomenon is that randomized questions prevent unwanted student collaboration (Figure 15, Table 1) the other (more benevolent) one is the unusual way of typing the degraded result. In the future, we have to have some new similar experiments when all students write their exams in the university building.

Find the regions of integration and evaluate the double integral over the region bounded by $y=4x$, $y=x$, $x=5$ and $x=9$, if

$$f(x,y) = x - 4y$$

The region is a normal region respect to
The shape of the region is:

(Click for List) ▾



The intervals of the integration are:

$$y_1 = \text{[input]} \leq y \leq \text{[input]} = y_2$$

$$x_1 = \text{[input]} \leq x \leq \text{[input]} = y_2$$

$$\int x - 4y \, dy = \text{[input]}$$

$$\int_{y_1}^{y_2} x - 4y \, dy = \text{[input]}$$

exact value please

$$\int_{x_1}^{x_2} \int_{y_1}^{y_2} x - 4y \, dy \, dx = \text{[input]}$$

Figure 14: One exam question with - and without TA

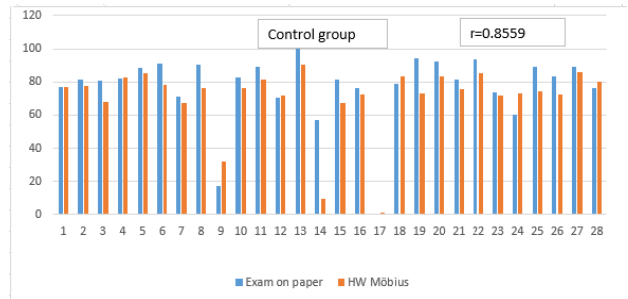
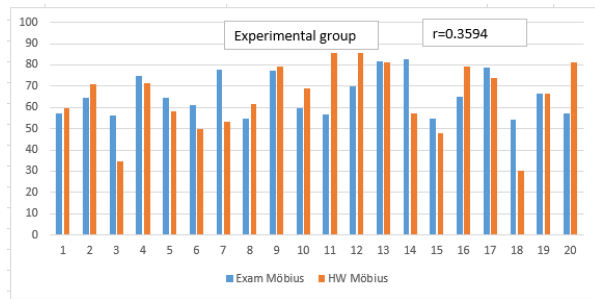


Figure 15: Exam results with - and without TA

	Civil-, Electrical Engineering	Computer Science Engineering
HW Möbius	70.48	64.79
Exam Paper	77	
Exam Möbius		66
correlation	0.8559	0.3594

Table 1: Result of experimental – and control group

A significant advantage of using the system from the professor's point of view is that it is much easier for lecturers to grade the tests and if there are a huge number of questions in the question banks, they can use them year-by-year. The system has many item statistics, which help the instructor correct the question hardness, form, and question types in the assignment.

9. Conclusion

After the above-mentioned examples, one possible answer to the question posed in the introduction is as follows.

What type of randomization is useful to introduce, and what are the limits of randomization?

Randomization in variables and figures is easy. For theoretical questions, the multiple-choice or list type is more usable. Sometimes self-made Maple packages are needed to avoid complicated calculations (e.g. Example 5.) Preparing the assignments, similar types of questions could be collected in a group, and the students get only one (or more) from each group. So not only the randomization of data but the randomization of questions increases the variability of assignments.

Why is it sometimes so dangerous, what are the benefits, and what are the limits of it?

Questions that can be easily divided into parts, but the response area can not handle if one answer is wrong and the user continues to calculate with this wrong value (e.g. Example 7).

Using randomized question banks is profitable for students because

- there is no transmitter language – in contrast to computer algebra systems
- it is based on cloud computing, for the user needs only a browser and a username
- it understands the mathematical equivalence
- they have prompt feedback
- they can practice without a time limit.
- visualization helps to understand the problem.

It is helpful for teachers because

- there are a lot of question types (open-ended and closed-ended questions)
- every student gets different questions
- there is flexible scoring, partial points
- they can create a different assessment for the same question bank easily
- it can follow up on the student's activity.
- there is no need to correct tests, and there is no photocopying.

Difficulties are liable to occur for students because

- there is no personal connection with the teacher
- mistype causes incorrect answer
- the software sometimes is not user friendly
- there is no cheating.

Some difficulties for teachers are that

- the process is time-consuming
- unconventional question definition is needed
- the developer has to get ready for all possible answer
- they need some knowledge not only in mathematics but in computer algebra and programming, as well
- the software sometimes is not user-friendly.

What type of math problems can be solved using random variables?

Asking definitions, conditions of theorems, and complex problem-solving problems can be randomized (naturally proof is not easy to randomize) (e.g. Example 3).

How can we ensure equal difficulty for the variation of questions?

Because similar difficulty is needed, the design sometimes does not allow using Maple randomization possibilities the backward thinking ensures a similar type and hardness (e. g. Example 6.).

Concluding a CAS-based assessment needs very careful planning, programming, and checking phases. In the planning period, it must be defined clearly, what part of the curriculum is important to ask, who are the target groups, and what is their background. Construct different assessments for different purposes (practicing, self-regulation, exam). Data acquisition is necessary, mainly the selection of the type of questions, splitting the exercises into parts, weighting, defining the questions clearly, and limiting randomization. We have to be careful that everyone gets a question of equal difficulty. In this part, the main guiding principle is the didactic of mathematics (naturally together with the correct mathematics).

During the programming section, close teamwork is necessary between the curriculum developer and IT expert (the best situation is if the members of the group are those math teachers, who are good in programming).

The check can take a long time. It must be teamwork to realize all possible sources of error.

Firstly the developers have to answer the questions in a different situation. Using the grading options in Möbius TA (class grades and item statistics) the developers could see which questions are easy and which are hard.

It seems to be a time-consuming process, but the work pays off. The same question bank could be used year by year, the teacher gets rid of boring test corrections. Using a well-prepared questionnaire (with detailed feedback) and after oral explanation, the role of the teacher is only to guide students, they can learn alone.

In the next future, our didactical research will focus on how these new techniques change the topics of mathematics curriculum in engineering education, and how will it help to focus better on deeper understanding instead of problem-solving schematically.

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